

Thermostatic properties of the Hagedorn Resonances

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In our recent works [1] it was shown that a system \mathcal{H} possessing a Hagedorn-like spectrum [2–5] ($C = \text{const}$)

$$\rho_{\mathcal{H}}(m) \approx C \exp(m/T_{\mathcal{H}}), \quad (1)$$

behaves like a perfect thermostat with the temperature $T_{\mathcal{H}}$, i.e., it imparts the Hagedorn temperature $T_{\mathcal{H}}$ to any other system to which \mathcal{H} is coupled. Such a property of the microcanonical system leads to the surprising behavior that the Laplace transform to the canonical ensemble

$$\mathcal{Z}(T) = \int_0^{\infty} \rho_{\mathcal{H}}(E) e^{-\frac{E}{T}} dE = C \frac{T_{\mathcal{H}} T}{T_{\mathcal{H}} - T}. \quad (2)$$

exists for any value of parameter $T < T_{\mathcal{H}}$ and diverges for $T = T_{\mathcal{H}}$. This fact led to the erroneous conclusion that the microcanonical temperature $T_{\mathcal{H}}$ is the limiting temperature, whereas it is *the only temperature* of the microcanonical system. Thus, we are facing the non-equivalence of the statistical ensembles for the exponential mass spectrum (1), which was noticed first by R. Carlitz [6], but was never discussed from the point of view of standard thermodynamics.

To elucidate the problem let us consider two systems A and B with level densities ρ_A and ρ_B , respectively. Let the systems be thermally coupled to each other with total energy E . We now calculate the distribution of this energy between the two systems,

$$\rho_T(x) = \rho_A(E - x)\rho_B(x) \quad (3)$$

Let A be a thermostat, i.e. $\rho_A(\epsilon) = e^{\epsilon/T_A}$. Then

$$\rho_T(x) = e^{(E - x)/T_A} \rho_B(x) = e^{E/T_A} e^{-x/T_A} \rho_B(x). \quad (4)$$

Let us integrate over x for macroscopic values of E

$$\int \rho_T(x) dx = e^{E/T_A} \int e^{-x/T_A} \rho_B(x) dx = e^{E/T_A} Z_B(T_A), \quad (5)$$

which gives rise to the partition function $Z_B(T_A)$ and the meaning of “implicit” thermostat. By changing “thermostat” we can change T_A and the temperature of B .

Thus, every time we construct a partition function, we imply the gedanken experiment of connecting the system to a thermostat, and that this experiment is actually possible for the system we are studying. Does this always work?

To see this, let us look for the most probable value of the distribution $\rho_T(x)$, which defines the equilibrium partition, by taking the logarithm and differentiating:

$$\begin{aligned} \ln \rho_T(x) &= \ln \rho_A(E - x) + \ln \rho_B(x) & (6) \\ \partial \ln \rho_T(x) / \partial x &= -\partial \ln \rho_A / \partial x + \partial \ln \rho_B / \partial x = 0 & \Leftrightarrow \\ &\Leftrightarrow 1/T_A = 1/T_B. & (7) \end{aligned}$$

For this to be possible, it is necessary that ρ_A and ρ_B admit the *same* logarithmic derivative somewhere in the allowed range of energy x .

Usually, and always for concave functions, $S(x) = \ln \rho(x)$ and $T = (\partial S / \partial x)^{-1}$ are such that $0 \leq T \leq \infty$. Thus, for such systems it is possible to match derivatives for whatever value of E . Thermal equilibrium is achievable over a broad range of temperatures.

However, if $S_A(E) = \ln \rho_A(E)$ is linear in E , then $T_A = (\partial S / \partial E)^{-1}$ is a constant, independent of E . In this case, it is up to B to look for the value of x at which its logarithmic derivative matches $1/T_A$. The system A is a thermostat at $T = T_A$ and the temperature of system B must assume the value $T_B = T_A$, if possible.

Now suppose that also $S_B(E) = \ln \rho_B(E)$ is linear in E with an inverse slope T_B . This means that only if $T_A = T_B$ is equilibrium possible, and the partition function of B , Z_B is meaningfully defined only for $T = T_B$ and not for $0 \leq T \leq T_B$. A temperature $T \neq T_B$ cannot be forced on a thermostat. It can only have its own intrinsic temperature T_B .

Placing systems A and B into contact will lead to a continuous heat flow from one system to the other. Thermal equilibrium is not achievable.

Summarizing: it is permissible to calculate a system’s partition function only if its $S(E)$ admits as inverse derivatives the very values we imposed through the Laplace transform. Failing that, the resulting partition function does not satisfy any thermodynamic criterion.

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